Benha University Faculty of Engineering – Shoubra Industrial Engineering Department Course: Mathematics 4 Code: EMP 202		Final Exam Date: May 12 , 2017 Duration Time : 2 hours Answer **All** questions
• The exam consists of one page		• No. of questions: 4 Total Mark: 40

Question 1

Solve the following equations: 12

(a) $xy\,dx + (1 + x^2)dy = 0$

(b) $(x + \sin y)dx + (y^2 + x\cos y)dy = 0$

(c) $y` - 2y = \frac{e^{2x}}{2x}$

(d) $y`` - 3y` - 4y = 4 + e^{3x}$

(e) $y`` - 4y = 2 + x^2$

(f) $(D^2 + 4)y = 1 + \cos^2 3x$

Question 2

(a)Find the L.T of the following:

(i) $f(t) = t^3 + \sinh 3t$ (ii) $f(t) = 2 + t\sin t$ (iii) $f(t) = e^{-2t}\cos 2t$ 3

(b)Find the inverse L.T of : (i) $F(s) = \frac{1}{(S-2)^4}$ (ii) $F(s) = \frac{s+3}{S^2-3s-4}$ 2

(c)By L.T, solve the equation : $y`` - 4y` + 4y = e^{2t}$, $y(0) = 0$ , $y`(0) = 1$. 5

Question 3

(a) Using the bisection method, find a root to the equation : $1 - x - \ln(x + 1) = 0$ in the interval [0, 1], number of iterations is 3. 4

(b)Find the line $y = a + bx$ and the curve $y = a + bx + cx^2$ that fits the data : (2, 3), (3, 5), (4, 6), (5, 12), (6, 14) 4

(c)Write the table of differences of the data: (1, 3), (3, 6), (5, 8), (7, 15). Also, find the value of y at x = 4. 4

Question 4

(a)Find the integral : $\int_0^\infty \frac{1}{\sqrt{2+x^4}}dx$ 4

(b)Find $f`(1)$ where $f(x) = \begin{cases} x^2, & x > 1 \\ 2^x - 1, & x \le 1 \end{cases}$ and h = 0.1 2

Good Luck *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a)The equation written as : $\frac{x}{1+x^2}\ dx + \frac{1}{y} dy = 0$

Its solution is : $\frac{1}{2}\ln(1+x^2) + \ln y = c$

(b)It is exact because $p_y = \cos y = q_x$.

Then $\int (x + \sin y)\mathrm{dx} = \frac{1}{2}x^2 + x\sin y$ and $\int (\mathrm{y}^2 + \mathrm{x}\ \cos \mathrm{y})\mathrm{dy} = \frac{1}{3}\mathrm{y}^3 + \mathrm{x}\sin \mathrm{y}$

Then the solution is : $\frac{1}{2}x^2 + x\sin y + \frac{1}{3}\mathrm{y}^3 = \mathrm{c}$

(c) It is linear. Then $\rho = \mathrm{e}^{\int -2\mathrm{dx}} = \mathrm{e}^{-2\mathrm{x}}$

Then the solution is : $y.\mathrm{e}^{-2\mathrm{x}} = \int \mathrm{e}^{-2\mathrm{x}}.\frac{e^{2x}}{2x}dx = \int \frac{1}{2\mathrm{x}}dx = \frac{1}{2}\ln x + c$

(d)The A.E is $m^2 - 3m - 4 = 0$. Then $m = -1, m = 4$

Then $\mathrm{y}_{\mathrm{c.f}} = A\ e^{-x} + B\ e^{4x}$

Also, $\mathrm{y}_{\mathrm{P.I}} = \frac{1}{D^2-3D-4}(4 + e^{3x}) = \frac{4}{0-0-4} + \frac{1}{9-9-4}e^{3x} = -1 - \frac{1}{4}e^{3x}$

The solution is : $y = \mathrm{y}_{\mathrm{c.f}} + \mathrm{y}_{\mathrm{P.I}}$

(e) The A.E is $m^2 - 4 = 0$. Then $m = -2, m = 2$

Then $\mathrm{y}_{\mathrm{c.f}} = A\ e^{-2x} + B\ e^{2x}$

Also, $\mathrm{y}_{\mathrm{P.I}} = \frac{1}{D^2-4}(2 + x^2) = \frac{1}{-4}\left(1 - \frac{1}{4}D^2\right)^{-1}(2 + x^2)$

$$= -\frac{1}{4}\left(1 + \frac{1}{4}D^2 + \frac{1}{16}D^4 + \cdots\right)(2 + x^2) = -\frac{1}{4}(2 + x^2 + \frac{1}{2})$$

The solution is : $y = \mathrm{y}_{\mathrm{c.f}} + \mathrm{y}_{\mathrm{P.I}}$

(f) The A.E is $m^2 + 4 = 0$. Then $m = 2i,\ m = -2i$

Then $y_{c.f} = A\cos 2x + B\sin 2x$

Also, $y_{P.I} = \frac{1}{D^2+4}(1 + cos^2 3x) = \frac{1}{D^2+4}\left(1 + \frac{1}{2}(1 + \cos 6x\right)$

$$= \frac{3/2}{0+4} + \frac{1}{-36+4}\frac{1}{2}\cos 6x = \frac{3}{8} - \frac{1}{64}\cos 6x$$

The solution is : $y = y_{c.f} + y_{P.I}$

---*12-Marks*

Answer of Question 2

(a)(i) $F(s) = \frac{3!}{s^4} + \frac{3}{s^2-9}$

(ii) $F(s) = \frac{2}{s} - \left(\frac{1}{s^2+1}\right)` = \frac{2}{s} + \frac{2s}{(s^2+1)^2}$

(iii) $F(s) = \frac{s+2}{(s+2)^2+4}$

---*3-Marks*

(b) (i) $f(t) = \frac{1}{3!}t^3.e^{2t}$

(ii) $F(s) = \frac{s+3}{S^2-3s-4} = \frac{A}{s-4} + \frac{B}{s+1} = \frac{7/5}{s-4} + \frac{-2/5}{s+1}$. Then $f(t) = \frac{7}{5}e^{4t} - \frac{2}{5}e^{-t}$

---*2-Marks*

(c)Since $L\{y`` - 4y` + 4y\} = L\{e^{2t}\}$, $y(0) = 0$, $y`(0) = 1$.

Then $(s^2Y - sy(0) - y`(0)) - 4(sY - y(0)) + 4Y = \frac{1}{s-2}$

From the given condition : $s^2Y - 0 - 1 - 4Y - 0 + 4Y = \frac{1}{s-2}$

Then $(s^2 - 4s + 4)Y = \frac{1}{s-2} + 1 = \frac{s-1}{s-2}$

Then $Y = \frac{s-1}{(s-2)(s^2-4s+4)} = \frac{s-1}{(s-2)^3} = \frac{1}{(s-2)^2} + \frac{1}{(s-2)^3}$

Then $y(t) = t.e^{2t} + \frac{1}{2}t^2.e^{2t}$

---*5-Marks*

Answer of Question 3

(a) f(x) = 1 – x – ln (x + 1) = 0

sign + + – – –

0 0.5 0.57 0.63 0.75 1

Iteration	[a, b]	x_n	$f(x_n)$	sign
0	[0, 1]	0.5	0.09	+
1	[0.5, 1]	0.75	– 0.3	–
2	[0.5, 0.75]	0.63	– 0.11	–
3	[0.5, 0.63]	0.57		

Then x* = 0.57

---*4-Marks*

(b) The line is : $y = a + bx = -3.6 + 2.9x$

And the parabolic curve is : $y = a + bx + cx^2 = 1.4 + 0.04x + 0.36x^2$

---*4-Marks*

(c)Since h = 2, then the table of finite differences is:

x	y	Δ	Δ^2	Δ^3
1	3			
		3		
3	6		– 1	
		2		6
5	8		5	
		7		
7	15			

The forward formula is:

$$y = f(x) = 3 + \frac{3}{1!.2}(x-1) + \frac{-1}{2!.4}(x-1)(x-3) + \frac{6}{3!.8}(x-1)(x-3)(x-5)$$

When x = 4, y = f(4) = 6.75

---*4-Marks*

Answer of Question 4

(a) $\int_0^\infty \frac{1}{\sqrt{2+x^4}}dx = \int_0^1 \frac{1}{\sqrt{2+x^4}}dx + \int_0^1 \frac{1}{\sqrt{1+2y^4}}dy = 0.68 + 0.88 = 1.56$

--*4-Marks*

(b)From $f(x) = \begin{cases} x^2, & x > 1 \\ 2^x - 1, & x \leq 1 \end{cases}$ and h = 0.1

Then $f`(1) = \frac{f(x+\Delta)-f(x-\Delta)}{2\Delta} = \frac{(1.1)^2-(2^{0.9}-1)}{2(0.1)} = 1.72$

--*2-Marks*

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